

## 5-5 Use Inequalities in a Triangle

Definition of Inequality--  $a > b$  iff there is a positive number  $c$ , such that  $a = b + c$

$$7 = 3 + 4$$

$$7 > 3$$

## Properties

Transitive

Addition/Subtraction

Multiplication/Division

Subst.

Ex:

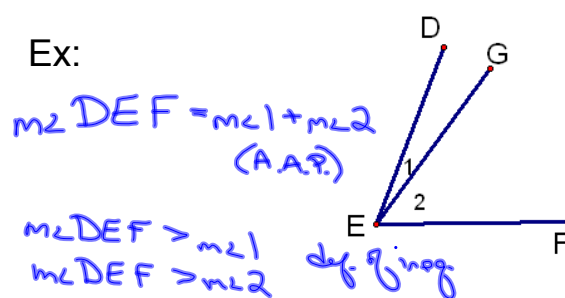


$$LN = LM + MN \quad \text{S.A.P.}$$

$$LN > LM \quad \text{def of seg}$$

$$LN > MN \quad \text{def of seg}$$

Ex:

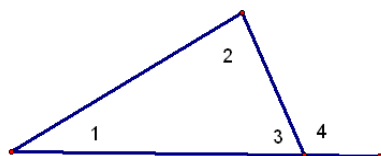


$$m\angle DEF = m\angle 1 + m\angle 2 \quad (\text{A.A.P.})$$

$$m\angle DEF > m\angle 1 \quad \text{def of ang}$$

$$m\angle DEF > m\angle 2 \quad \text{def of ang}$$

Ex:



$$m\angle 4 = m\angle 1 + m\angle 2 \quad \text{Ext } \angle \text{ thm}$$

$$m\angle 4 > m\angle 1$$

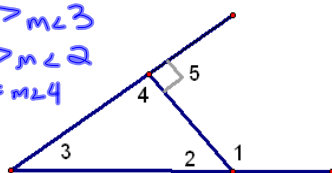
$$m\angle 4 > m\angle 2$$

Exterior Angle Inequality Theorem—The exterior angle of a triangle is greater than either of its corresponding remote interior angles

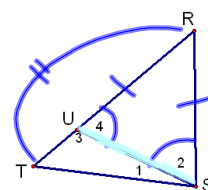
Which angle is the largest?

(Figure is not drawn to scale.)

$$\begin{aligned} m\angle 5 &> m\angle 3 \\ m\angle 5 &> m\angle 2 \\ m\angle 5 &= m\angle 4 \end{aligned}$$



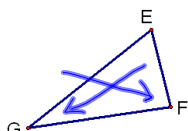
$$\begin{aligned} m\angle 1 &> m\angle 3 \\ m\angle 1 &> m\angle 4 \end{aligned}$$

Given:  $\triangle RST$  $RU = RS$  $RT > RS$ Prove:  $m\angle RST > m\angle T$ 

- |  |                               |
|--|-------------------------------|
| ①  | ① Given                       |
| ② $\angle 4 \cong \angle 2$                          | ② B.A.T                       |
| ③ $m\angle 4 \cong m\angle 2$                        | ③ def of $\cong$              |
| ④ $m\angle 4 > m\angle 1$<br>$m\angle 4 > m\angle T$ | ④ Ext $\angle$ Inequality thm |
| ⑤ $m\angle RST = m\angle 4$                          | ⑤ A.A.P                       |
| ⑥ $m\angle 2 > m\angle T$                            | ⑥ Subst.                      |
| ⑦ $m\angle RST > m\angle 2$                          | ⑦ def of ineq                 |
| ⑧ $m\angle RST > m\angle T$                          | ⑧ transitive                  |

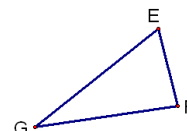
Theorem 5.10--If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

G:  $GE > EF$   
 C:  $m\angle F > m\angle G$



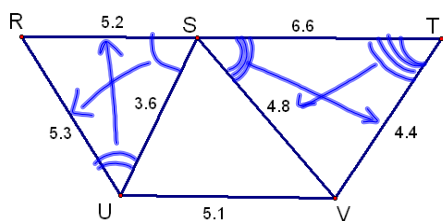
Theorem 5.11--If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

G:  $m\angle F > m\angle G$   
 C:  $GE > EF$



1. Which is greater,  $m\angle RSU$  or  $m\angle SUR$ ?

2. Which is greater,  $m\angle TSV$  or  $m\angle STV$ ?



Determine which angle has the greatest measure.

17.  $\angle 1, \angle 2, \angle 4$

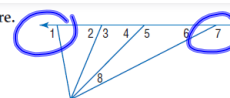
18.  $\angle 2, \angle 4, \angle 6$

19.  $\angle 3, \angle 5, \angle 7$

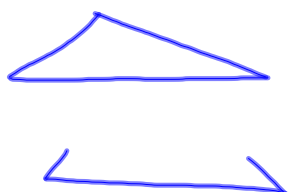
20.  $\angle 1, \angle 2, \angle 6$

21.  $\angle 5, \angle 7, \angle 8$

22.  $\angle 2, \angle 6, \angle 8$



Pasta Challenge

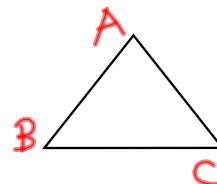


Thm. 5.12--The triangle inequality theorem--the sum of the lengths of any 2 sides of a triangle is greater than the length of the 3rd side.

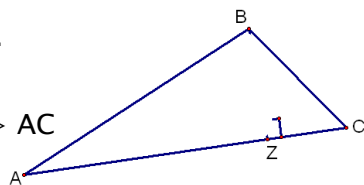
$$AB + BC > AC$$

$$BC + AC > AB$$

$$AB + AC > BC$$



Let's Prove it.

Given:  $\triangle ABC$ Prove:  $AB + BC > AC$ 

Do the lengths represent a triangle?

$$4, 5, 7 \quad \text{yes} \quad 4 + 5 > 7 \checkmark$$

$$13, 12, 20 \quad \text{yes}$$

$$7, 14, 21 \quad \text{no} \quad 7 + 14 = 21$$

$$7, 7, 7 \quad \text{yes}$$

$$8, 8, 19 \quad \text{no}$$

Two sides of a triangle are 6 and 11.  
What is the range of the 3rd side?

$$5 < x < 17$$

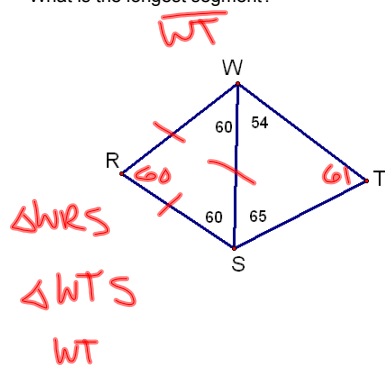
$$\begin{array}{r} 11 \\ -6 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 11 \\ +6 \\ \hline 17 \end{array}$$

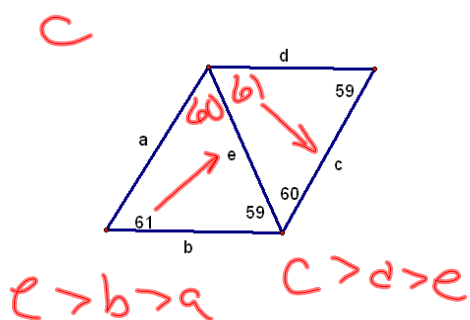
Two sides of a triangle are 12 and 18.  
What is the range of the 3rd side?

$$6 < x < 30$$

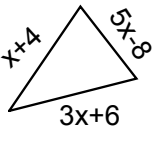
What is the longest segment?



What is the longest segment?



Describe the possible values for x.



Handwritten work for the triangle inequality problem:

$$\begin{aligned}
 &3x+6 > 0 \\
 &x > -2 \\
 &x > -4 \\
 &5x-8 > 0 \\
 &x > \frac{8}{5} \\
 &5x-8+3x+6 > x+4 \\
 &8x-2 > x+4 \\
 &7x > 6 \\
 &x > \frac{6}{7} \\
 &x+4+5x-8 > 3x+6 \\
 &x > \frac{10}{3} \\
 &x+4+3x+6 > 5x-8 \quad x < 18
 \end{aligned}$$

Final inequality:  $\frac{10}{3} < x < 18$

HW p331-332

#s 6, 10-13, 16-19, 21-23, 30, 31, 33, 34